# Cost Reduction using Passenger Centric Timetabling Technical report 

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#### Abstract

Summary This technical report is part of the project entitled Cost Reduction using Passenger Centric Timetabling. The project is a collaboration between the Swiss Federal Railways and the Transport and Mobility (Transp-OR) laboratory of EPFL. In this report, we present the status of the research project after 6 months of investigation. Starting from the mathematical optimization model of the timetable design proposed by Robenek (2016), the first challenge was to scale up the methodology for the Swiss railway network. To meet this challenge, we propose a mathematical model that can design a timetable for the Swiss passenger service, while taking into account (explicitly) the behavior of the passengers. The result of the model is the timetable itself as well as the routing of the passengers. Several KPIs can be obtained using post-processing on the results: train-km, passenger-km, load factor, etc. This report is organized as follows. In Section 1, we briefly describe the problem at hand and we present (1) the SBB data we received and (2) the path generation algorithm we developed for this project. In Section 2, we present the mathematical model scaled up for the Swiss railway case. Results of the MILP are showed in Section 3 for two different small instances. Section 4 contains the current work on the heuristic method. Finally, Section 5 lists shows the next steps.


## 1 Problem at hand, data, and path generation algorithm

Problem at hand The passenger centric train timetabling problem is a multiobjective problem that can be graphically summarized as:


This problem is inter-disciplinary. It combines the discrete choice theory, that models the passengers' behavior, and operations research, that decides on the departure times of the trains (i.e. the timetable). The attributes affecting the passengers' choices with respect to the operated timetable are quantified into a single variable of passenger generalized travel time. The objective of the proposed model is the trade-off between the profit of the train operating company and the overall travel time of the passengers.

Data We use the RER Vaud as case study for this project.


The area extends to the following terminus stations:
\& Allaman,
? Vallorbe,
: Grandson/Yverdon,
P Payerne,
\& Puidoux,
? Aigle.
Note : the RER extension Payerne to Morat was not included in the sub area as this branch, which is part of the RER system today, will not be served by RER in the STEP 2030 reference scenario.

This corresponds to the following lines:

| LSA | AIG | VAL | AIG | GRS | CU | ALL | ALL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| COS | ROC | DAY | VIL | YV | LTY | ETOY | ETOY |
| BY | VIL | BRT | TER | EP | PU | STP | STP |
| REN | VEY | CR | MX | ESP | LS | MOR | MOR |
| PRMA | TER | AX | CL | CHV | PRMA | STJ | STJ |
| LS | MX | LSA | BURI | BAV | REN | DEN | LON |
| PU | CL | COS | TOUR | ECL | BY | REN | REN |
| LTY | BURI | BY | VV | COS | VU | PRMA | PRMA |
| VTE | TOUR | REN | STSA | VU | COS | LS | LS |
| CU | VV | PRMA | RIV | BY | ECL | PUN | PUN |
| EPS | STSA | LS | EPS | REN | BAV | CVN | CVN |
| RIV | RIV | PU | CU | PRMA | CHV | GRV | BOSS |
| STSA | EPS | LTY | VTE | LS | ESP | PUI | GRV |
| VV | CU | VTE | LTY | PU | EP | MRL | PUI |
| TOUR | VTE | CU | PU | LTY | YV | PAL | PAL |
| BURI | LTY | EPS | LS | CU | GRS |  |  |
| CL | PU | RIV | PRMA |  |  |  |  |
| MX | LS | STSA | REN |  |  |  |  |
| TER | PRMA | VV | BY |  |  |  |  |
| VEY | REN | TOUR | COS |  |  |  |  |
| VIL | BY | BURI | LSA |  |  |  |  |
| ROC | COS | CL | AX |  |  |  |  |
| AIG | LSA | MX | CR |  |  |  |  |
|  |  | TER | BRT |  |  |  |  |
|  |  | VIL | DAY |  |  |  |  |
|  |  | AIG | VAL |  |  |  |  |


| PAL | PAL | LS | PAY | LS | CN | PUI | VV |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| MRL | PUI | PUI | GM | PAL | PAY | CHX | VVFU |
| PUI | GRV | PAL | LUC | MD | LUC | CORS | CORS |
| GRV | BOSS | PALV | MD | LUC | MD | VVFU | CHX |
| CVN | CVN | CHAT | ECU | PAY | PAL | VV | PUI |
| PUN | PUN | ECU | CHAT | CN | LS |  |  |
| LS | LS | MD | PALV |  |  |  |  |
| PRMA | PRMA | LUC | PAL |  |  |  |  |
| REN | REN | GM | PUI |  |  |  |  |
| DEN | LON | PAY | LS |  |  |  |  |
| STJ | STJ |  |  |  |  |  |  |
| MOR | MOR |  |  |  |  |  |  |
| STP | STP |  |  |  |  |  |  |
| ETOY | ETOY |  |  |  |  |  |  |
| ALL | ALL |  |  |  |  |  |  |

Concerning the demand, from the origin and destination matrix received from SBB, we started by matching the OD zones to the OD stations. We finally have 1531 OD pairs. The graphic below illustrates the demand. We can see that the most important flows are for the stations 2157 and 2548, that represent respectively Lausanne and Renens.


The OD matrix also includes the time of day distribution. The demand is divided in $\mathbf{1 4 4}$ time slots, each representing 10 minutes of the day. The computation of the time distribution is based on the empirical time distribution
in SIMBA (2030-STEP-Reference modeling) and the time distributions for the sub area have been computed as follows:
\& Type 1: Internal OD pairs: original time distribution
? Type 2: OD pairs which are cut at one end: average over all OD pairs with the same trip end in the sub area

8 Type 3: OD pairs with both trip ends outside of the subarea: average over all OD pairs of Type 2 for these cordon zones

The demand by TOD is illustrated on the Figure below. As we can see, there is a first peak early in the morning and a second peak early in the evening.


Some basic statistics are shown in the Table below:

|  | Total demand | \# OD $i$ | \# Group $(i, t)$ |
| :--- | :---: | :---: | :---: |
| SBB raw data | $71,486.56$ | 3,969 | 571,536 |
| Keep if demand $>0$ | $71,486.56$ | 1,534 | 114,703 |
| Keep if $\mathrm{O} \neq \mathrm{D}$ | $71,343.32$ | 1,531 | 114,508 |
| Keep if $D>10$ | $22,021.37$ | 459 | 1,136 |

## Path generation algorithm



> Iteration\#1:
> $\mathrm{o}-\mathrm{s}$
> Iteration\#2:
> $\mathrm{o}-\mathrm{s}-\mathrm{t}$
> $\mathrm{o}-\mathrm{s}-\mathrm{u}$
> $\mathrm{o}-\mathrm{s}-\mathrm{v}$
> Iteration\#3:
> $\mathrm{o}-\mathrm{s}-\mathrm{t}-\mathrm{x}$
> $\mathrm{o}-\mathrm{s}-\mathrm{t}-\mathrm{y}$
> $\mathrm{o}-\mathrm{s}-\mathrm{u}-\mathrm{d}$
> $\mathrm{o}-\mathrm{s}-\mathrm{v}-\mathrm{d}$
> Iteration\#4:
> $\mathrm{o}-\mathrm{s}-\mathrm{t}-\mathrm{x}-\mathrm{d}$
> $\mathrm{o}-\mathrm{s}-\mathrm{t}-\mathrm{y}-\mathrm{d}$
> $\mathrm{o}-\mathrm{s}-\mathrm{u}-\mathrm{d}$
> $\mathrm{o}-\mathrm{s}-\mathrm{v}-\mathrm{d}$

When constructing the paths, we check the following conditions
? Never two times the same station
? Never two times the same line
? No change of line if the next station of the new line is the same as the next station of the current line


## 2 Mathematical model

Sets The mathematical model is based on the following sets of parameters:

- $I$ is the set of origin-destination pairs,
- $T_{i}$ is the set of preferred departure times for OD pair $i$,
- $P_{i}$ is the set of possible paths for OD pair $i$,
- $L$ is the set of operated lines,
- $L^{p}$ is the set of lines in the path $p$,
- $V^{\ell}$ is the set of available trains for the line $\ell$
- $S^{\ell}$ is the set of segments on line $\ell$
- $S^{\ell p}$ is the set of segments on line $\ell$ used by path $p$


## Main parameters

- $n_{i}^{t}$ : number of passengers who wish to travel between origin-destination pair $i$ at preferred departure time $t$,
- $a_{i}^{t}$ : preferred departure time of passenger group $(i, t)$ to her destination (in minutes past midnight),
- $e$ : revenue rate (CHF / Pax-km)
- $f$ : operating cost (staff cost (wages etc), vehicle cost, network usage fees (including overhead cost)) per train in single traction (in CHF/Train-km),
- o: operating cost per additional train unit, in double or triple traction (in CHF/Train-km),
- $k_{\ell}$ : length of line $\ell($ in km$)$,
- $h_{s}$ : length of segment $s$ (in km),
- $g$ : maximum length of a train (in train units),
- $q$ : capacity of a single train unit,
- c: cycle duration (in minutes).
- $m$ : minimum transfer time (in minutes).
- $\ell_{1}^{p}$ : first line used in path $p$

We denote $(i, t)$ the group of passengers who wish to travel between origindestination pair $i$ at preferred departure time $t$.

## Decision variables

- $x_{i}^{\text {tp }} \in\{0,1\}$ : takes value 1 if the passenger group $(i, t)$ chooses path $p$, and 0 otherwise,
- $y_{i}^{\text {tplv }} \in\{0,1\}$ : takes value 1 if the passenger group $(i, t)$ using path $p$ takes train $v$ on the line $\ell$, and 0 otherwise,
- $d_{v}^{\ell} \in \mathbb{N}$ : denotes the departure time of $\operatorname{train} v$ on the line $\ell$,
- $z_{v}^{\ell} \in \mathbb{N} \backslash\{0\}$ : is an integer variable used to model the cyclicity corresponding to train $v$ on line $\ell$,
- $\omega_{v s}^{\ell} \in \mathbb{N}$ : denotes the number of passengers on segment $s$ in train $v$ on the line $\ell$,
- $\mu_{v}^{\ell} \in \mathbb{N}$ : denotes the number of train units of train $v$ on the line $\ell$,
- $\alpha_{v}^{\ell} \in\{0,1\}$ : takes value 1 if train $v$ on the line $\ell$ is being operated, and 0 otherwise.

Objective function The objective function ensures that total profits of the operator are maximized:

$$
\begin{equation*}
\max \sum_{\ell \in L} \sum_{v \in V^{\ell}} \sum_{s \in S^{\ell}} \omega_{v s}^{\ell} \cdot e \cdot h_{s}-\sum_{\ell \in L} \sum_{v \in V^{\ell}}\left(\alpha_{v}^{\ell} \cdot f \cdot k_{\ell}+\left(\mu_{v}^{\ell}-\alpha_{v}^{\ell}\right) \cdot o \cdot k_{\ell}\right) \tag{1}
\end{equation*}
$$

The first part of the objective function computes the revenues obtained from the sale of train tickets to the passengers and the second part deduced the costs that are supported by the operator.

Constraints In this section the supply-side constraints of the mathematical model are listed and they are then explained one by one.

$$
\begin{array}{lr}
\sum_{p \in P_{i}} x_{i}^{t p} \leq 1, & \forall i \in I, \forall t \in T_{i} . \\
\sum_{v \in V^{\ell}} y_{i}^{t p \ell \ell}=x_{i}^{t p}, & \forall i \in I, \quad \forall t \in T_{i}, \quad \forall p \in P_{i}, \forall \ell \in L^{p} . \\
\left(d_{v}^{\ell}-d_{v-1}^{\ell}\right)=c \cdot z_{v}^{\ell}, & \forall \ell \in L, \forall v \in V^{\ell}: v>1 . \\
d_{v}^{\ell} \leq d_{v+1}^{\ell}-1, & \forall \ell \in L, \forall v \in V^{\ell}: v<\left|V^{\ell}\right| . \\
\omega_{v s}^{\ell}=\sum_{i \in I} \sum_{t \in T_{i}} \sum_{p \in P_{i}: l \in L^{p}} y_{i}^{t p \ell v} \cdot n_{i}^{t}, & \forall \ell \in L, \forall v \in V^{\ell}, \forall s \in S^{\ell p} . \\
\mu_{v}^{\ell} \cdot q \geq \omega_{v s}^{\ell}, & \forall \ell \in L, \forall v \in V^{\ell}, \forall s \in S^{\ell} . \\
\alpha_{v}^{\ell} \cdot g \geq \mu_{v}^{\ell}, & \forall \ell \in L, \forall v \in V^{\ell} . \\
x_{i}^{t p} \in\{0,1\}, & \forall i \in I, \forall t \in T_{i}, \forall p \in P_{i} . \\
y_{i}^{t p \ell v} \in\{0,1\}, & \forall i \in I, \\
\omega_{v s}^{\ell} \in \mathbb{N}, & \forall t \in T_{i}, \\
d_{v}^{\ell} \in \mathbb{N}, z_{v}^{\ell} \in \mathbb{N} \backslash\{0\}, \mu_{v}^{\ell} \in P_{i}, \forall \ell \in L^{p}, \forall v \in V^{\ell} . \\
& \forall \ell \in L, \forall v \in V^{\ell}, \forall s \in S^{\ell} . \\
& \forall \ell \in L, \forall v \in V^{\ell} .
\end{array}
$$

- Constraint (2) ensures that every passenger group (and thus every passenger) is using at most one path to get from her origin to destination.
- Constraint (3) ensures that every passenger group (and thus every passenger) takes exactly one train on each of the lines constituting the chosen path.
- Constraint (4) ensures the cyclicity. It ensures that two consecutive trains are separated by a multiple of $c$ minutes.
- Constraint (5) ensures the departure of the trains are in ascending order with a difference of at least one minute.
- Constraint (6) determines the number of passengers on each segment for each train of each line.
- Constraint (7) determines the number of units used for each train of each line, based on the number of passengers and the capacity of a single train unit.
- Constraint (88) ensures that the maximum train length is not exceeded and determines if a train on a line is operated or not.
- Constraint (9)-(12) are the domain constraints.

The next variables and constraints relate to the allocation of passengers to the different path of the network. The general idea is each group of passengers $(i, t)$ want to select the path that minimizes his generalized travel time.

The generalized travel time for each passenger group $(i, t)$ and each path $p$, can be expressed as:

$$
\begin{equation*}
\tau_{i}^{t p}=\sum_{\ell \in L^{p}} r_{i}^{p \ell}+\beta_{W} \cdot w_{i}^{t p}+\beta_{T} \cdot\left(\left|L^{p}\right|-1\right)+\beta_{E} \cdot \delta_{i}^{t p}+\beta_{L} \cdot \gamma_{i}^{t p} \tag{13}
\end{equation*}
$$

where

- $\beta_{W} \geq 0, \beta_{T} \geq 0, \beta_{E} \geq 0, \beta_{L} \geq 0$ are preference coefficients or weights for waiting time, number of transfers, lateness and earliness,
- $r_{i}^{p \ell}$ is the running time for OD pair $i$ on path $p$ using line $\ell$,
- $w_{i}^{t p}$ is the waiting time for passenger group $(i, t)$ using path $p$,
- $\left(\left|L^{p}\right|-1\right)$ is the number of transfers associated with path $p$,
- $\delta_{i}^{t p}$ is the scheduled delay of being early for passenger group ( $i, t$ ) using path $p$,
- $\gamma_{i}^{t p}$ is the scheduled delay of being late for passenger group $(i, t)$ using path $p$,

The schedule delay variables, for being early or late, are computed as follows:

$$
\begin{array}{ll}
\delta_{i}^{t p} \geq\left(a_{i}^{t}-\left(d_{v}^{\ell_{1}^{p}}+b_{i}^{p p_{1}^{p}}\right)\right)-M \cdot\left(1-y_{i}^{t p L_{1}^{p} v}\right), \quad \forall i \in I, \quad \forall t \in T_{i}, \forall p \in P_{i}, \forall v \in V^{\ell_{1}^{p}} . \\
\gamma_{i}^{t p} \geq\left(\left(d_{v}^{\ell_{1}^{p}}+b_{i}^{p \ell_{1}^{p}}\right)-a_{i}^{t}\right)-M \cdot\left(1-y_{i}^{t p L_{1}^{p} v}\right), \quad \forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}, \forall v \in V^{\ell_{1}^{p}} . \tag{15}
\end{array}
$$

$$
\begin{equation*}
\delta_{i}^{t p}, \gamma_{i}^{t p} \geq 0 \tag{16}
\end{equation*}
$$

$$
\forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}
$$

where

- $b_{i}^{p \ell}$ is the time to arrive from the starting station of the line $\ell$ to the origin/transferring point of the OD pair $i$ in the path $p$.
- $d_{v}^{\ell_{1}^{p}}$ is the departure time of train $v$ for the first line used in the path $p$.
- $\left(d_{v}^{\ell_{1}^{p}}+b_{i}^{p \ell_{1}^{p}}\right)$ is the departure time for passenger group ( $i, t$ ) using path $p$.

The waiting time for each group of passengers $(i, t)$ who selects the path $p$ is given by the sum of all waiting times happening on this path. The waiting time for each change of line is computed as:
$w_{i}^{t p \ell_{1}} \geq\left(\left(d_{v_{1}}^{\ell_{1}}+b_{i}^{p \ell_{1}}\right)-\left(d_{v_{2}}^{\ell_{2}}+b_{i}^{p \ell_{2}}+r_{i}^{p \ell_{2}}+m\right)\right)-M \cdot\left(1-y_{i}^{t p \ell_{2} v_{2}}\right)-M \cdot\left(1-y_{i}^{t p \ell_{1} v_{1}}\right)$, $\forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}, \forall \ell_{1}, \ell_{2} \in L^{p}: \ell_{1}>1, \ell_{2}=\ell_{1}-1, \forall v_{1} \in V^{\ell_{1}}, \forall v_{2} \in V^{\ell_{2}}$.
$w_{i}^{t p \ell_{1}} \leq\left(\left(d_{v_{1}}^{\ell_{1}}+b_{i}^{p \ell_{1}}\right)-\left(d_{v_{2}}^{\ell_{2}}+b_{i}^{p \ell_{2}}+r_{i}^{p \ell_{2}}+m\right)\right)+M \cdot\left(1-y_{i}^{t p \ell_{2} v_{2}}\right)+M \cdot\left(1-y_{i}^{t p \ell_{1} v_{1}}\right)$, $\forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}, \forall \ell_{1}, \ell_{2} \in L^{p}: \ell_{1}>1, \ell_{2}=\ell_{1}-1, \forall v_{1} \in V^{\ell_{1}}, \forall v_{2} \in V^{\ell_{2}}$.
$w_{i}^{t p}=\sum_{\ell \in L^{p} \backslash\{1\}} w_{i}^{t p \ell}, \quad \forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}$.
$w_{i}^{t p}, w_{i}^{t p \ell} \geq 0, \quad \forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}, \forall \ell \in L^{p} . \backslash\{1\}$
where $M$ is a big number.
Due to limited capacity, it is possible that not all passengers are served within the planned horizon. The generalized travel time for each passenger group $(i, t)$ that cannot be served is calculated as follows:

$$
\begin{equation*}
\nu_{i}^{t}=\min _{p \in P_{i}}\left(\sum_{l \in L^{p}} r_{i}^{p \ell}\right)+\beta_{B} \cdot c+\beta_{T} \cdot u_{i}+\beta_{L} \cdot\left(1440+c-a_{i}^{t}\right) \tag{21}
\end{equation*}
$$

where $u_{i}$ is the number of transfers in the shortest path for OD $i$.

This generalized travel time represents the option of taking the first possible shortest path outside of the planning horizon (1440 minutes).

The generalized travel time for each passenger group $(i, t)$, denoted $\tau_{i}^{t}$, is then computed as:

$$
\begin{equation*}
\tau_{i}^{t} \geq \tau_{i}^{t p}-M \cdot\left(1-x_{i}^{t p}\right), \quad \forall i \in I, \quad \forall t \in T_{i}, \forall p \in P_{i} \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\tau_{i}^{t} \geq \nu_{i}^{t} \cdot\left(1-\sum_{p \in P_{i}} x_{i}^{t p}\right), \quad \forall i \in I, \forall t \in T_{i} \tag{23}
\end{equation*}
$$

The generalized travel time, $\tau_{i}^{t}$, is in minutes.
If we denote by $V O T$ the value of time, i.e. the willingness-to-pay for travel time savings, we obtain the generalized travel cost as follows:

$$
\begin{equation*}
\mathbb{C}_{i}^{t}=V O T \cdot \tau_{i}^{t}, \quad \forall i \in I, \quad \forall t \in T_{i} \tag{24}
\end{equation*}
$$

The total generalized travel cost, defined as the sum of the generalized travel cost for all passengers, is therefore computed as:

$$
\begin{equation*}
G T C=V O T \sum_{i \in I} \sum_{t \in T_{i}} n_{i}^{t} \cdot \tau_{i}^{t} . \tag{25}
\end{equation*}
$$

Epsilon constraint will be used to ensure that the total generalized travel cost does not exceed a certain level.

$$
\begin{equation*}
V O T \sum_{i \in I} \sum_{t \in T_{i}} n_{i}^{t} \cdot \tau_{i}^{t} \leq \epsilon \cdot U B \tag{26}
\end{equation*}
$$

where $U B$ is an upper bound for the GTC and $\epsilon$ a factor that can vary between 0.1 and 1 .

## 3 Pareto frontier and preliminary results on small instance

The Pareto Frontier is the set of all Pareto efficient allocations between two instances, which correspond to optimal values of both instance in a certain environment. In this case, we will be defining our Pareto Frontier, between revenues and passengers' Generalized Travel Time (GTT) by running our cyclic model for two different sets of input data.

Unfortunately the large dimension of the mathematical model does not allow us to test the model on the full set of data (RER Vaud). For this reason, we decided to test the mathematical model using two small instances: OBS3 and OBS4. The first one consists of all the OD pairs departing between 7:35 am and 8:05 am, with a demand restricted to at least 10 . ODS4 consisted of all the OD pairs departing between 7:05 am and 8:05 am, with a demand restricted to at least 10. The running time was set to 3600 seconds (1h) for OBS3, whereas OBS4 ran for 10800 seconds (3h). The steps for each set of simulations were as follow:
? First, we run the cyclic model, where we maximize revenues while excluding Constraint (26). It means that there is no limit for the Generalized Travel Time, GTT.
\& After running the first simulation, we reach the fixed time limit. The GTT value calculated will be considered as the upper bound parameter value for the next steps. Indeed, by excluding Constraint (26), the GTT obtained corresponds to the worst-case scenario for the passengers.

8 Then, we run the model but including Constraint (26). Different values of $\epsilon$ will be simulated. For OBS3, we simulated values for $\epsilon$ between 0.5 and 1 , with a 0.1 pace, whereas for OBS4 we simulated values for $\epsilon$ between 0.55 and 1 , with a 0.05 pace.

After running all simulation mentioned above, we extract the following results: the best know solution with its corresponding GTC, the running time, the value for $\epsilon$ as well as the optimality gap (only for OBS4 case).

OBS3_3600 The results are shown in the table below:

| Simulation set No 1 |  |  |  |
| ---: | :---: | :---: | :---: |
| $\mathbf{R}$ | GTT | t | $\boldsymbol{\varepsilon}$ |
| 4081 | 854134 | 3600 | 1.00 |
| 3659 | 768721 | 3600 | 0.90 |
| 3771 | 683308 | 3600 | 0.80 |
| 2815 | 597894 | 3600 | 0.70 |
| 3875 | 512481 | 3600 | 0.60 |
| -24256 | 427067 | 3600 | 0.50 |

Based on these results, we were able to get the Pareto Frontier shown in the graph below

## Pareto Frontier - Simulation set № 1



OBS4_10800 The results are shown in the table below:

| Simulation set No 2 |  |  |  |  |  |
| ---: | :---: | :---: | :---: | ---: | :---: |
| $\mathbf{R}$ | GTT | t | $\boldsymbol{\varepsilon}$ | Gap |  |
| 7713 | 2375369 | 10800 | 1.00 | $82.97 \%$ |  |
| 9676 | 2256601 | 10800 | 0.95 | $45.85 \%$ |  |
| 10854 | 2137832 | 10800 | 0.90 | $30.02 \%$ |  |
| 10843 | 2019064 | 10800 | 0.85 | $30.15 \%$ |  |
| 10377 | 1900296 | 10800 | 0.80 | $35.99 \%$ |  |
| 8856 | 1781527 | 10800 | 0.75 | $59.35 \%$ |  |
| 11002 | 1662759 | 10800 | 0.70 | $28.26 \%$ |  |
| -15229 | 1543990 | 10800 | 0.65 | $192.66 \%$ |  |
| -58705 | 1425222 | 10800 | 0.60 | $124.04 \%$ |  |
| -71987 | 1306453 | 10800 | 0.55 | $119.60 \%$ |  |

Based on these results, we were able to get the Pareto Frontier shown in the graph below

Pareto Frontier - Simulation set № 2


We can see that both Pareto Frontier, obtained through different sets, have similar shapes. Another important point to note is that for certain values of $\epsilon$, profits do not vary. This means that for different level of passenger satisfaction, represented by the Generalized Travel Cost, the revenues are almost the same. This confirms the results obtained in the PhD thesis of Robenek (2016).

## 4 Variable neighborhood search heuristic

In the previous section, we have seen that the preliminary results seem to confirm the results obtained by Robenek (2016). The analysis, based on very small instances using RER Vaud data, shows that an improvement of passenger satisfaction while maintaining a low profit loss for the railway company can be achieved. However, as expected, the model is too large to be solved exactly using the full data set. We are therefore investigating an heuristic method. A neighborhood search heuristic will be investigated. A neighborhood search heuristic explores the large set of feasible solutions by using an exploration tool that generates a sequence of solutions (potentially all of them). Such a tool is called a neighborhood structure. In our case, each type of timetable (cyclic, non-cyclic, and hybrid) will have a different neighborhood structure. The passenger assignment will be carried out within the search algorithm. The passenger assignment procedure is described in Algorithm 1.

The passenger assignment algorithm is currently implemented. Once the passenger assignment algorithm will be validated, we will move to the next step, which is the implementation of the neighborhood search. The general pseudo code for a neighborhood search is illustrated in Algorithm 2. The code will be scaled up to our specific problem.

```
Algorithm 1
    procedure PASSENGER ASSIGNMENT
    Input
    -The set of operated lines ( \(L\) )
    -The set of available trains per line \(\ell\left(V^{\ell}\right)\)
    -The set of origin-destination pairs ( \(I\) )
    -The set of preferred departure time for origin-destination \(i\left(T_{i}\right)\)
    -The set of possible paths for origin-destination \(i\left(P_{i}\right)\)
    -The set of lines used in path \(p\left(L^{p}\right)\)
    -The first line used in path \(p\left(\ell_{1}^{p}\right)\)
    -The set of segments on line \(\ell\left(S^{\ell}\right)\)
    -The set of segments on line \(\ell\) used by path \(p\left(S^{\ell p}\right)\)
    -The departure time for each train \(v\) and line \(\ell\left(d_{v}^{\ell}\right)\)
    -The number of train units for each train \(v\) and line \(\ell\left(\mu_{v}^{\ell}\right)\)
    -The revenue rate in CHF per Pax-km (e)
    -The operating cost per train in single traction in CHF per Train-km \((f)\)
    -The operating cost per additional train unit in CHF per Train-km (o)
    -The length of line \(l\left(k_{\ell}\right)\)
    -The length of segment \(s\left(h_{s}\right)\)
    -The capacity of a single train unit \((q)\)
    -The minimum transfer time ( \(m\) )
    -The preference coefficients for waiting time, number of transfers, lateness,
    and earliness \(\left(\beta_{W}, \beta_{T}, \beta_{E}, \beta_{L}\right)\)
    -The running time for origin-destination \(i\) on path \(p\) using line \(\ell\left(r_{i}^{p \ell}\right)\)
    -The time to arrive from the starting station on line \(\ell\) to the ori-
    gin/transferring point of the origin-destination pair \(i\) on path \(p\left(b_{i}^{p \ell}\right)\)
    -A list of passengers, their origin-destination, and preferred departure time
    ( \(a_{i}^{t}\) )
    Output
    -The number of passengers on segment \(s\) in train \(v\) on line \(\ell\)
    -The generalized cost of travel
    -The total profits of the operator
        for \(i \in I\) do
        for \(t \in T_{i}\) do
            for \(p \in P_{i}\) do
                for all combination of trains and lines do
                        calculate GTC
            sort travel options according to GTC ascending
            if first travel option doesn't violate capacity of any train then
                assign passenger to this option
                update the total GTC
```

```
Algorithm 2
    1 Objective
    2 Find a good feasible solution of the optimization problem \(\min _{x} f(x)\)
        subject to \(x \in \mathcal{F}\).
    Input
        The objective function \(\mathrm{f}: \mathbb{R}^{n} \rightarrow \mathbb{R}\)
        The feasible set \(\mathcal{F}\)
        An initial feasible solution \(x_{0} \in \mathcal{F}\)
        A neighborhood structure N
    Output
        A feasible solution \(\chi^{*}\)
    Repeat
        \(x_{c}:=\operatorname{argmin}_{x \in \mathrm{~N}\left(x_{k}\right) \cup \mathcal{F}} f(x)\)
        if \(f\left(x_{c}\right)<f\left(x_{k}\right)\) then
            \(x_{k+1}:=x_{c}\)
            \(k:=k+1\)
15 Until \(f\left(\chi_{c}\right)=f\left(x_{k}\right)\) or \(N\left(x_{k}\right) \cup \mathcal{F}=\emptyset\)
\(16 \chi^{*}:=\chi_{k}\)
```


## 5 Next steps

As explained in the previous sections, the model is too large to be solved exactly in the full data set. The next step is therefore to investigate the heuristic presented in Section 4. Then, timetables obtained with the heuristic will be compared to the ones received from SBB , and conclusions will be drawn.

## Next steps

\% Neighborhood search heuristic:

- Implementation of the passenger allocation algorithm (in progress)
- Definition of neighborhood structures based on the type of timetables (in progress)
- Implementation of the neighborhood search (to be done)
- Adding capacity as a decision in the framework (to be done)
\& Testing (in progress)
- Analysis of results and feedback from SBB
- Comparison with existing timetables

8 Conclusions for the RER Vaud (to be done)

